Assimilating subsurface hydrology data

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Introduction

The vertical distribution of subsurface moisture and its accessibility for evapotranspiration is a key determinant of the fate of ecosystems and their feedback on the climate system.

- **Motivation:** The hydrologically active soil column in CLM4 [1], has a globally uniform thickness of **3.8**m. This unrealistic critical assumption is a deficiency of the model and requires further attention in the future development of the model.
- **Goal:** Ultimately, we are interested in developing algorithms to model deep subsurface water dynamics along with its interaction with deep root systems.

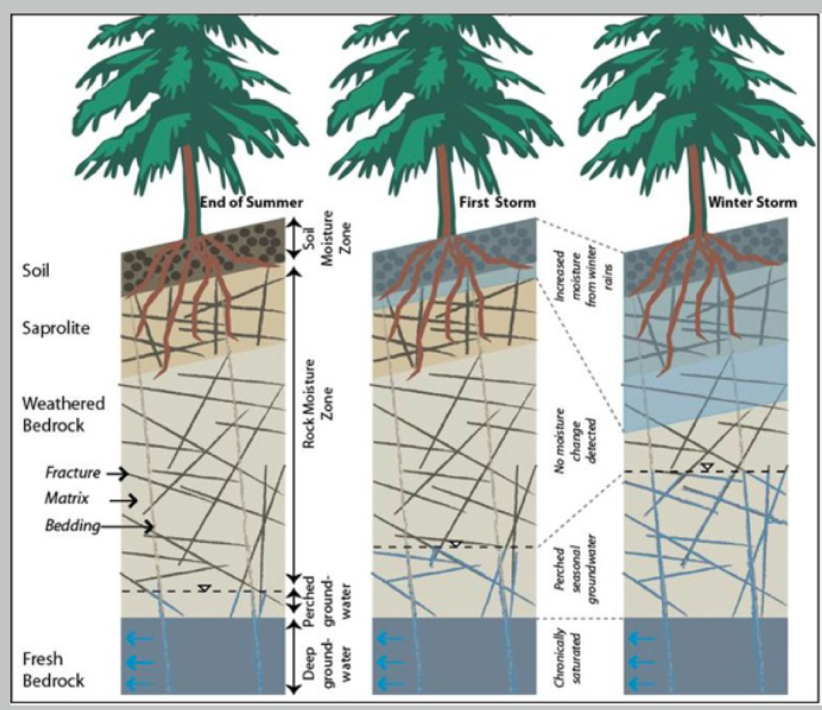


Figure : (1) Schematic representation of the proposed vertical structure. The heterogeneous subsurface is divided into three layers (soil, saprolite and weathered bedrock), with varying depths.

Richards' Equation

Richards' PDE describes the movement of liquids in unsaturated porous media [2]. It appears mainly in two forms:

- θ-based:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \psi}{\partial z} - K(\theta)\right] + S(z,t)\ , \qquad (1)$$

- ψ-based:

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\right] + S(z,t)\ , \qquad (2)$$

where the description of the variables is as follows:

Name	Description	Dimensions
$\theta(z,t)$	soil moisture (water content)	$[L^3/L^3]$
$\psi(z,t)$	pressure head (suction)	[L]
z	vertical space dimension	[L]
t	time dimension	[T]
$C(\psi) \equiv \frac{d\theta}{d\psi}$	specific moisture capacity	[1/L]
$D(\theta) \equiv K(\theta)/C(\theta)$	unsaturated diffusivity	$[L^2/T]$
$K(\theta)$	unsaturated hydraulic conductivity	[L/T]
$S(z,t)$	source/sink term	[1/T]

- Boundary Conditions: For the experiments that follow, we applied zero flux conditions both at the top ($z = 0$) and at the bottom ($z = z_{tot}$) of the equation, i.e. : $K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right) = 0$
- Main parameter of interest here is the **hydraulic conductivity** $K(\theta)$, which varies with the volumetric water content θ [3].

Datasets: Berkeley HydroWatch Project

- Located on a small (**4000** m^2) steep (**35**°) hill-slope nicknamed 'Rivendell' in the Angelo Coast Range Reserve in the Eel River Watershed, in Northern California, the *Berkeley HydroWatch Project* has drilled 12 wells and uses more than **800** sensors to record the hydrologic status in real time.
- High frequency (less than **30** minutes) data, for nearly five years, shows that the water tables, roughly **18** meters below the surface, can respond in less than **8** hours to the first rains, suggesting very fast flow through macro-pores and fractured rock (e.g. Figure 1).

Data assimilation methodology - EnKF/S

- To assimilate the water table data obtained from the *Berkeley HydroWatch Project*, with the model equations (Richards' PDE) we use an ensemble Kalman filter [4].
- To reduce the sampling error caused by remote locations in the state vector we propose to use a Gaussian correlation function with local support, (e.g. $\rho(i,j;\lambda) = exp\{-\frac{(i-j)^2}{\lambda^2}\}$).
- For estimating the desired parameters of the model we will use a RTS ensemble Kalman smoother [5].

Experiments with artifical data

To demonstrate this approach, we apply the algorithm on synthetic data. Numerical solution of the original PDE (ψ-based), with zero flux boundary conditions, produces the "truth", which provides the **water table depth** observations every **30** min. The following table summarizes the setting:

Name :	z_{tot}	Δ_z	T_{tot}	Δ_t	θ_r	θ_s	K_{sat}	N_{ens}
Value :	500 cm	1 cm	240 hr	30 min	0.1	0.5	20 cm/hr	60

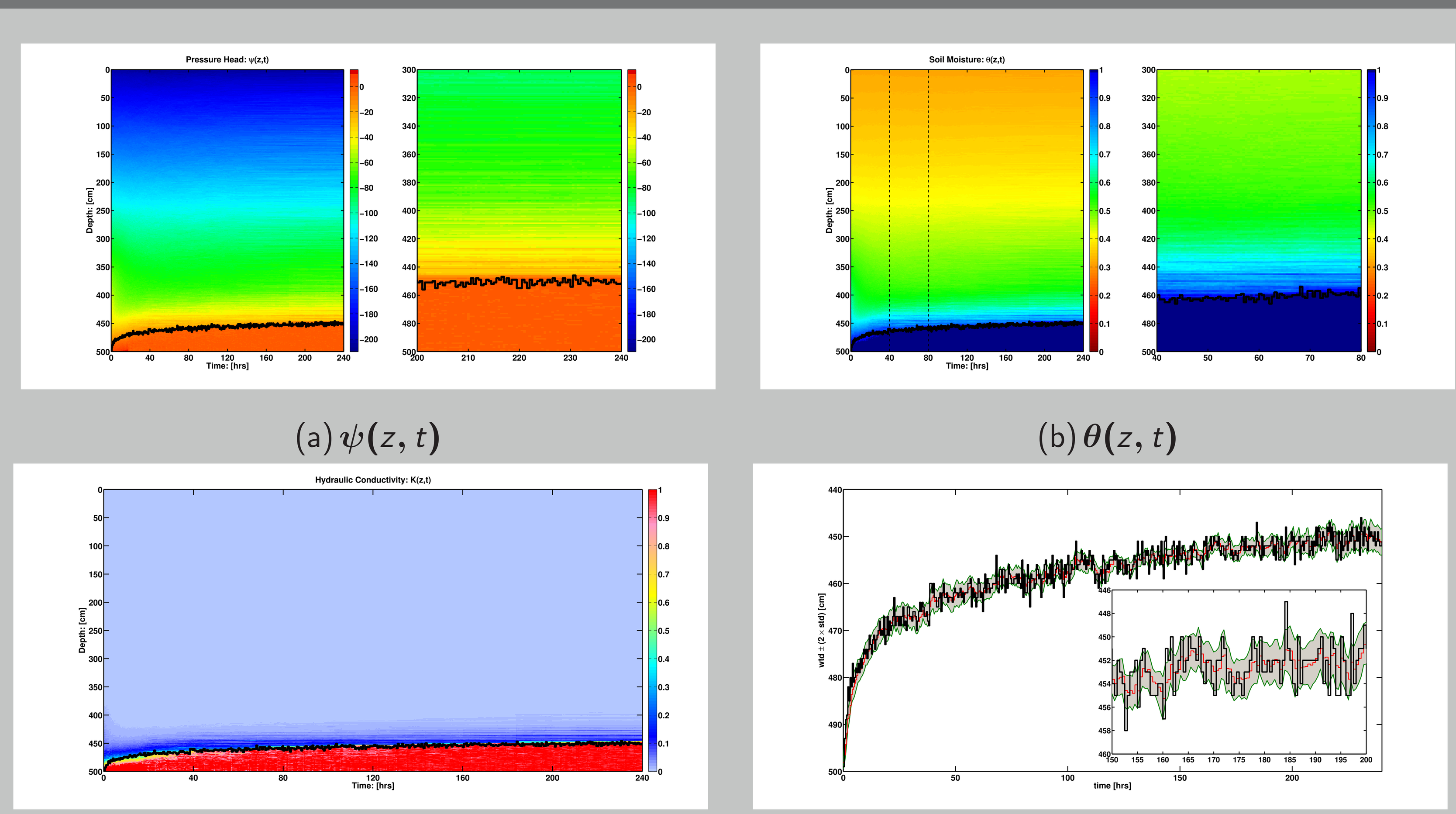
The soil-water content θ as a function of the pressure head ψ, and the hydraulic conductivity K as a function of θ are given by [3]:

$$\theta(\psi) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|\psi|)^n]^m}\ , \qquad (3)$$

$$K(\Theta) = K_{sat}\sqrt{\Theta}[1 - (1 - \Theta^{1/m})^m]^2\ , \text{with } \Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}\ , \qquad (4)$$

where $\alpha = 0.0335$ and $m = 1 - 1/n$, with $n = 2$.

Results (preliminary)



(a) $\psi(z,t)$ (b) $\theta(z,t)$ (c) $K(z,t)$ (d) wtd. estimation vs (noisy) observations

Figure : 2(a) shows the pressure head ψ, as a function of depth z and time t. Figures 2(b) and 2(c) the same but for soil-moisture θ and the hydraulic conductivity K respectively. On all plots the **wtd** (black solid line) is superimposed for contrast with the estimated values. 2(d) shows the mean wtd value (solid red line), surrounded by two times std (grey shaded area), as a function of time. Again the black solid line indicates the true values of the **wtd**.

Future work

This work is ongoing and there are several directions in which we are looking to move forward:

1. Currently the hydraulic conductivity $K(\theta)$ is given by a fixed formula, see Eq.(5). However, to capture the effect of fractures in the weather bedrock layer we will include a random term (which we will attempt to estimate).
2. To complete the model we need to add a lateral conductivity term such as $K_{Lat}(z) = \alpha_L \times K(z)$, i.e. provide the water the means to escape from the 1-D model and account for the slow decline of the water table depth.

References

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