A new stochastic hydraulic conductivity approach for modeling one-dimensional vertical flow in variably saturated porous media

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Introduction

Observations in coastal Northern California show that water table, some 20m below the surface, rises by as much as one meter within days after the first winter rains suggesting preferential flow paths and water storage in rock fractures that may sustain transpiration through summer dry seasons.

- **Motivation:** The hydrologically active soil column in CLM4.5 [1], has a globally uniform thickness of 3.8m. This unrealistic critical assumption is a deficiency of the model and requires further attention in the future development of the model.
- **Goal:** Ultimately, we are interested in developing algorithms to model deep subsurface water dynamics along with its interaction with deep root systems.

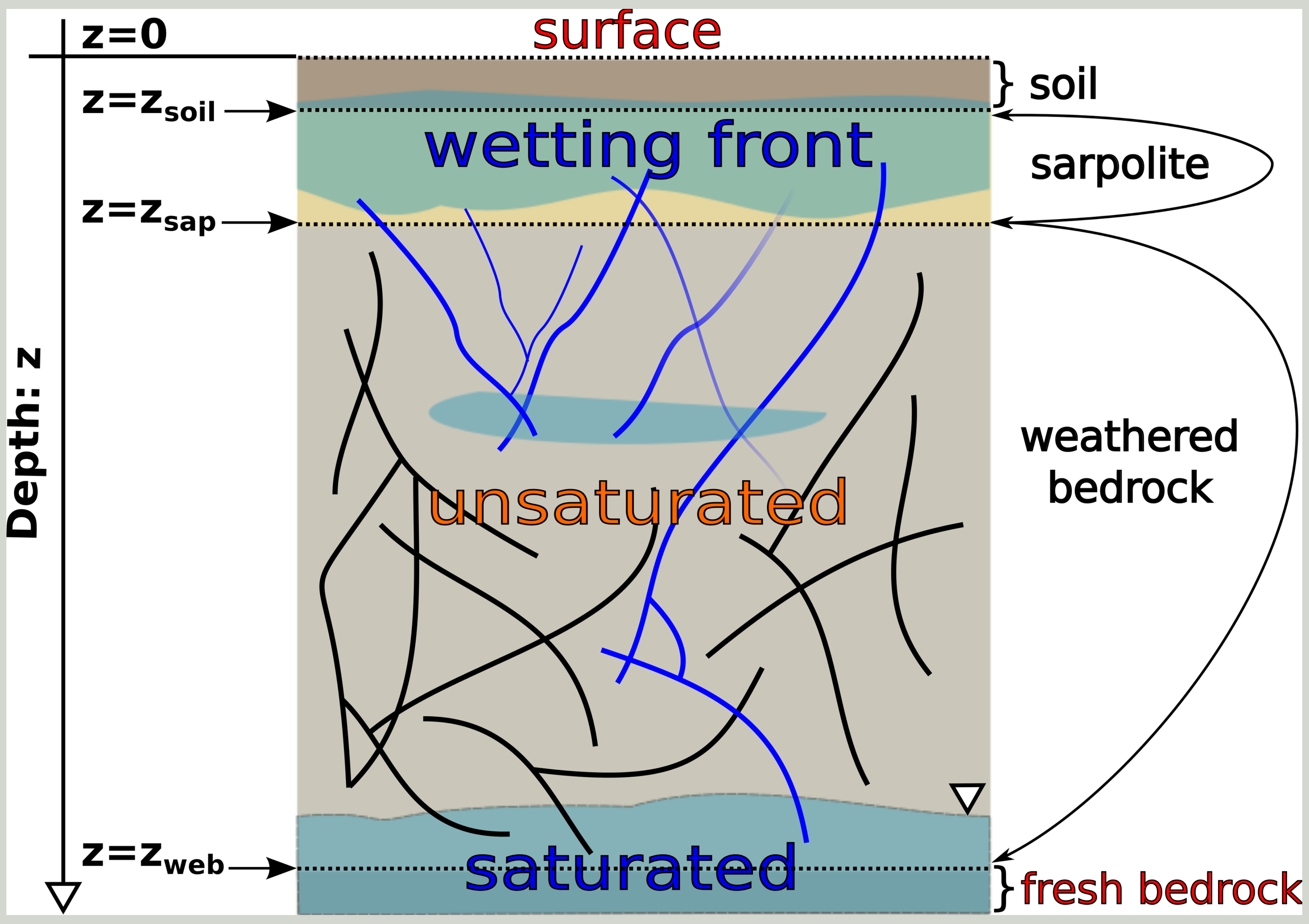

Figure : (1) Illustration of the proposed vertical structure. The heterogeneous subsurface is divided into three virtual layers (soil, saprolite and weathered bedrock), with varying depths.

Governing Equations

Richards' PDE describes the movement of liquids in unsaturated porous media [2]. The pressure head ψ(z,t) form of this equation is given by:

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\right] + S(z,\psi) \ , \qquad (1)$$

where z is the vertical space dimension [L], t is the time dimension [T], ψ(z,t) is the pressure head (suction) [L], C(ψ) is the specific moisture capacity [1/L], K(ψ) is the unsaturated hydraulic conductivity [L/T] and S(z, ψ) is a sink term [1/T]. In our setting [L: cm] and [T: hrs].

- Initial Conditions: $\psi(z, t = 0) = \psi_0 \in \mathbb{R}^D$,
- Top Boundary: $K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\Big|_{z=0} = \min(q_{rain}(t), 0)$,
- Bottom Boundary: $K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\Big|_{z=z_{fb}} = 0$, for $t \geq 0$.

The sink term is defined as:

$$S(z,\psi) = \begin{cases} -\alpha_L \psi(z,t), & z \in [z_{\hat{w}}, z_{\triangledown}] \\ 0, & \text{otherwise} \end{cases} \ , \qquad (2)$$

where $\alpha_L = 1.5 \times 10^{-3}$, $z_{\hat{w}}$ is the *estimated* and $z_{\triangledown}$ the *observed* water table depths respectively. The water retention curve is given by:

$$\theta(\psi) = \theta_{ret} + (\theta_{sat} - \theta_{ret})[1 + (\alpha|\psi|)^n]^{-m} \ , \qquad (3)$$

with $\alpha = 0.0335$, $m = 0.5$, $n = 2$ and θ_{sat}, θ_{ret} are the saturation and retention values of soil moisture respectively.

Hydraulic Conductivity

The main parameter of interest here is the **hydraulic conductivity** K(ψ). Traditional (deterministic) approaches for modeling this crucial parameter include the well known van Genuchten model [3]:

$$K(\Theta) = K_{sat}\sqrt{\Theta}[1 - (1 - \Theta^{1/m})^m]^2 \text{ , with } \Theta = \frac{\theta - \theta_{ret}}{\theta_{sat} - \theta_{ret}} \ . \qquad (4)$$

However this model is very slow in capturing the fast response of the water table, with dry initial conditions. Instead in this work we propose a new **stochastic** model [4]:

$$K(\Theta) = \Theta^{\lambda} K_{rnd} \text{ , with } K_{rnd} \sim \text{LogN}(\nu, \Lambda) \ , \qquad (5)$$

with $\Theta \in [0, 1]$ the effective saturation and $\lambda > 0$ a tuning parameter.

Example of K(Θ)

K_{rnd} is defined such as: $\langle K_{rnd} \rangle = \mu(z)$ and $\text{Var}[K_{rnd}] = \Sigma(1 - \Theta)$, where the angle brackets $\langle \cdot \rangle$ denote expected value, the noise amplitude $\Sigma \in \mathbb{R}^+$ and $\mu(z) \geq 0, \forall\, z \in [0, z_{web}]$, to mimic the effect of the naturally occurring heterogeneity in the underground [5].

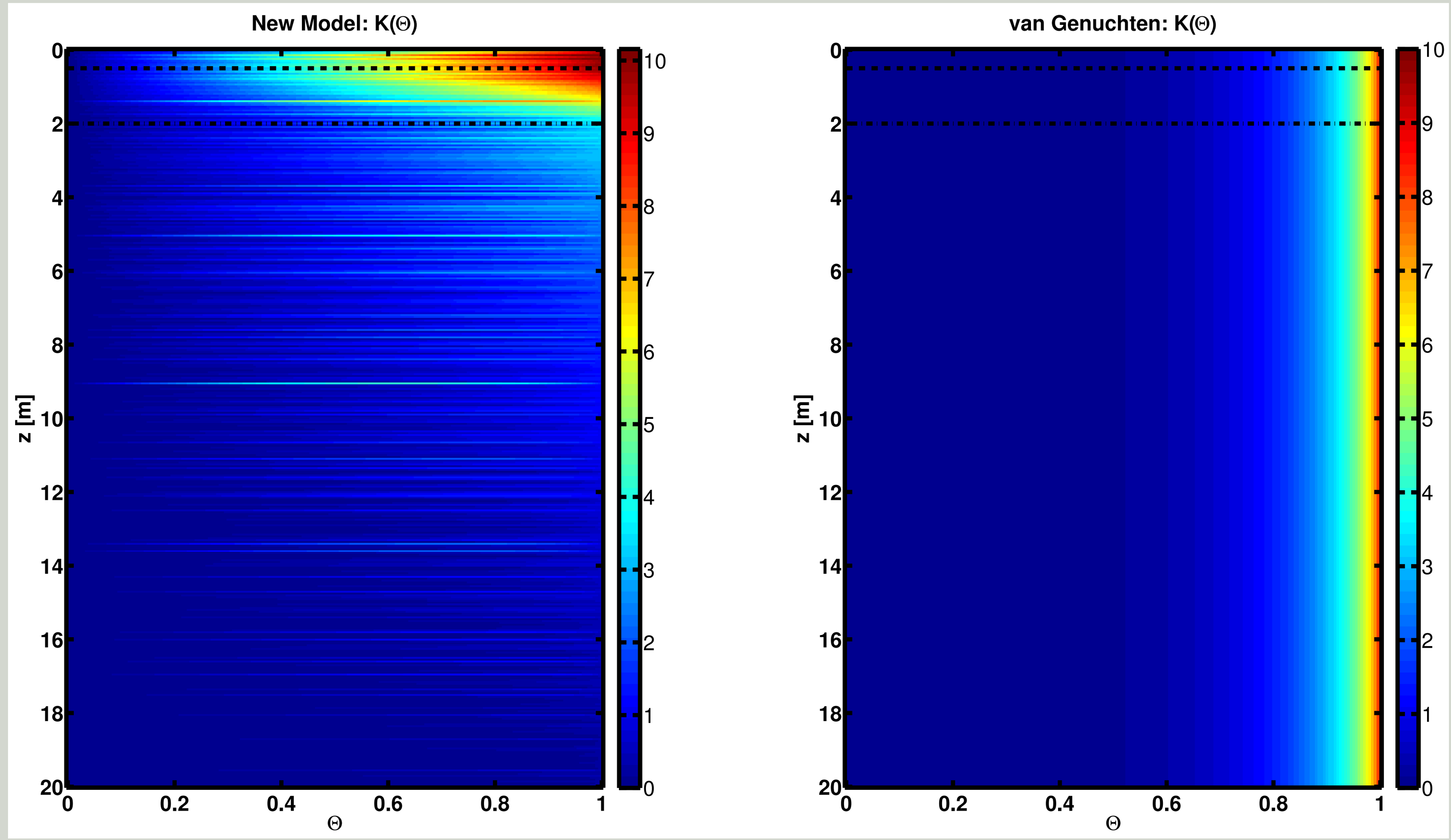

Figure : (2) Comparison of the new stochastic K(Θ) (left panel), from a single noise realization with $\Sigma = 2$ and $\lambda = 1$, with the van Genuchten hydraulic conductivity (right panel), with $K_{sat} = 10$ and $m = 0.5$. Both plots are presented as functions of depth **z** and normalized soil moisture Θ. Dashed lines mark the end of soil and saprolite layers.

Results

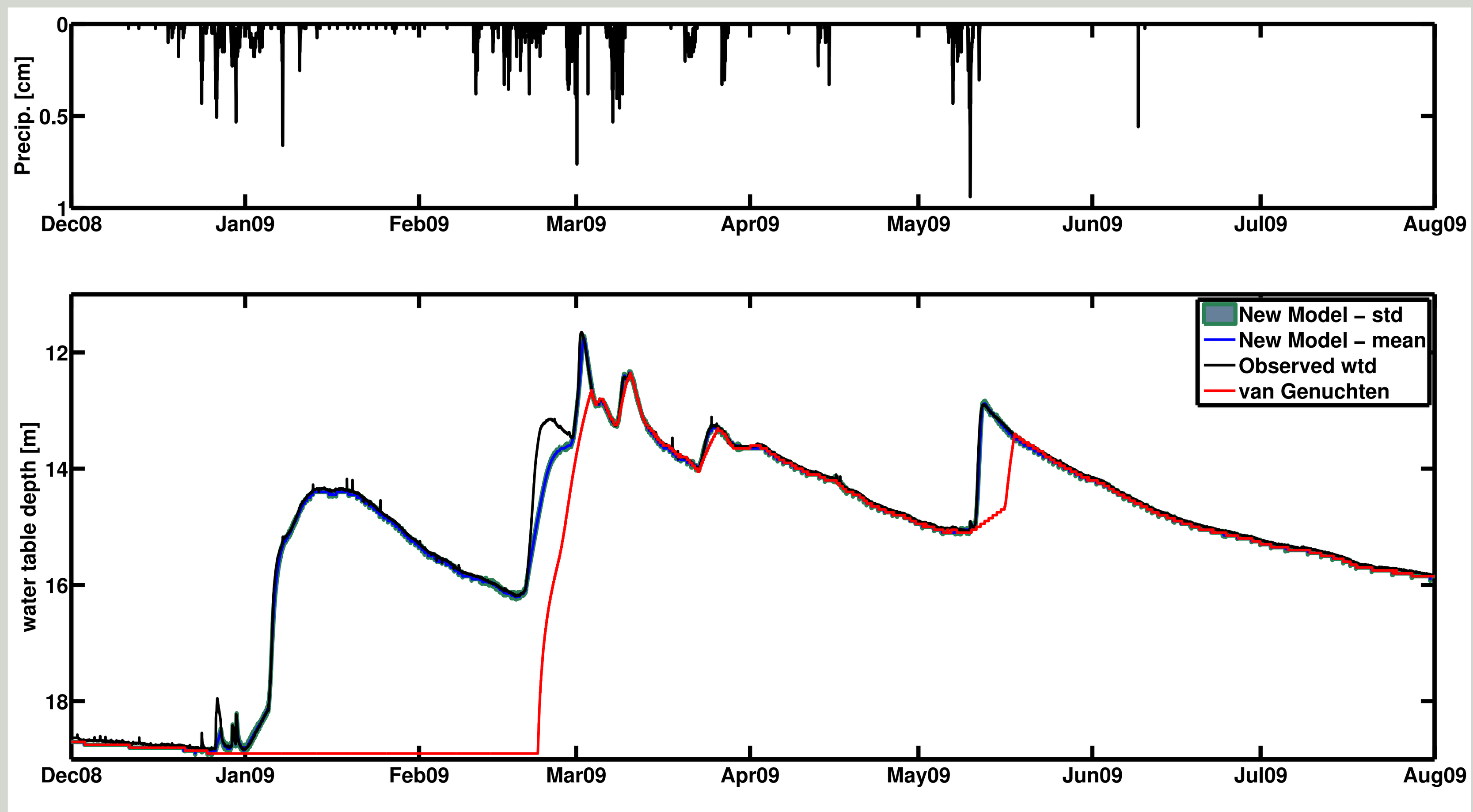

Figure : (3) Top panel shows the precipitation data for the WY:08–09. Bottom panel contrasts the results of the current study with mean (blue line) and standard deviation (green shaded area), with the result of the deterministic van Genuchten (red line), against the observed water table depth values (black line) of well No.10, for the same dates.

Results show that the stochastic hydraulic model (Eq. 5) can capture the rapid response of the water table depth to rainfall, with greater accuracy than the deterministic approach.

Future Work

Initial results are encouraging and the next steps include testing this new stochastic approach on data from other sites and different time periods.

Acknowledgements

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References

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